Power System Analysis

Chapter 11 Power System Operation

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Outline

1. Overview
2. Unit commitment
3. Optimal dispatch
4. Frequency control
5. System security

Overview

Central challenge

Balance supply & demand second-by-second

- While satisfying operational constraints, e.g. injection/voltage/line limits
- Unlike usual commodities, electricity cannot (yet) be stored in large quantity

Overview

Traditional approach

Bulk generators generate 80% of electricity in US (2020)

- Fossil (gas, coal): 60%, nuclear: 20%

They are fully dispatchable and centrally controlled

- ISO determines in advance how much each generates when & where

They mostly determine dynamics and stability of entire network

- System frequency, voltages, prices

Overview

Traditional approach

Challenges

- Large startup/shutdown time and cost
- Uncertainty in future demand (depends mostly on weather)
- Contingency events such as generator/transmission outages

Elaborate electricity markets and hierarchical control

- Schedule generators and determine wholesale prices
- Day-ahead (12-36 hrs in advance): unit commitment
- Real-time (5-15 mins in advance): economic dispatch
- Ancillary services (secs - hours): frequency control, reserves

Overview

Future challenges

Sharply increased uncertainty makes balancing more difficult

- Renewable sources such as wind and solar
- Random large frequent fluctuations in net load, e.g., Duck Curve due to PV
- Contingency events such as generator/transmission outages
- *Response*: real-time feedback control, better monitoring & forecast, stochastic OPF

Low-inertia system

- Bulk generators have large inertia that is bedrock of stability
- They will be replaced by inverter-based resources with low or zero inertia, e.g., PV
- *Response*: dynamics and stability need to be re-thought

Indispatchable renewable generation resources

- *Response*: More active dynamic feedback control of flexible loads to match fluctuating supply

Overview

Optimal power flow

Unit commitment and economic dispatch can be formulated as OPF

- OPF underlies many (other) power system applications
- State estimation, stability and security analysis, volt/var control, demand response

Constrained optimization

$$\min_{u,x} \quad c(u,x) \quad \text{s.t.} \quad f(u,x) = 0, \quad g(u,x) \leq 0$$

- Optimization vars: control u, network state x
- Cost function: $c(u,x)$
- Constraint functions: $f(u,x)$, $g(u,x)$
- They depend on the application under study

Outline

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Unit commitment

Solved by ISO in day-ahead market 12-36 hrs in advance

- Determine which generators will be on (commitment) and their output levels (dispatch)
- For each hour (or half hour) over 24-hour period
- Commitment decisions are binding
- Dispatch decisions may be binding or advisory

Two-stage optimization

- Determine commitment, based on assumption that dispatch will be optimized

Unit commitment

Problem formulation

Model

- Network: graph $G = (\overline{N}, E)$
- Time horizon: $T := \{1,2,\ldots,T\}$, e.g., t = 1 hour, $T = 24$

Optimization vars

- Control:
    - Commitment: on/off status $\kappa(t) := \left(\kappa_j(t), j \in \overline{N}\right), \ \kappa_j(t) \in \{0,1\}$
    - Dispatch: real & reactive power injections $u(t) := (u_j(t), j \in \overline{N})$
- Network state:
    - Voltages $V(t) := (V_j(t), j \in \overline{N})$
    - Line flows $S(t) := \left(S_{jk}(t), S_{kj}(t), (j,k) \in E\right)$

Unit commitment

Problem formulation

Capacity limits: injection is bounded if it is turned on

$$\underline{u}_j(t)\kappa_j(t) \quad \leq \quad u_j(t) \quad \leq \quad \overline{u}_j(t)\kappa_j(t)$$

Startup and shutdown incur costs regardless of injection level

$$d_{jt}(\kappa_j(t-1), \kappa_j(t)) = \begin{cases} \text{startup cost} & \text{if} \quad \kappa_j(t) - \kappa_j(t-1) = 1 \\ \text{shutdown cost} & \text{if} \quad \kappa_j(t) - \kappa_j(t-1) = -1 \\ 0 & \text{if} \quad \kappa_j(t) - \kappa_j(t-1) = 0 \end{cases}$$

UC problems in practice includes other features

- Once turned on/off, bulk generator stays in same state for minimum period

Unit commitment

Problem formulation

Two-stage optimization

$$\min_{\kappa\in\{0,1\}^{(N+1)T}} \quad \sum_t \sum_j d_{jt}\left(\kappa_j(t-1), \kappa_j(t)\right) \quad + \quad c^*(\kappa)$$

where $c^*(\kappa)$ is optimal dispatch cost over entire horizon T:

$$c^*(\kappa) \quad := \quad \min_{(u,x)} \quad \sum_t c_t(u(t), x(t); \kappa(t))$$

$$\text{s.t.} \quad f_t(u(t), x(t); \kappa(t)) = 0, \quad g_t(u(t), x(t); \kappa(t)) \le 0, \quad t \in T$$

$$\tilde{f}(u, x) = 0, \quad \tilde{g}(u, x) \le 0$$

- Each time t constraint includes injection limits
- $\tilde{f}(u, x) = 0, \quad \tilde{g}(u, x) \le 0$ can include ramp rate limits

Unit commitment

Problem formulation

UC in practice

- Binary variable makes UC computationally difficult for large networks
- Typically use linear model, e.g., DC power flow, and solve mixed integer linear program

Serious effort underway in R&D community to scale UC solution with AC model

- e.g., ARPA-E Grid Optimization Competition Challenge 2

Outline

1. Overview
2. Unit commitment
3. Optimal dispatch
   - OPF formulation
   - Imbalance and error model
4. Frequency control
5. System security

Optimal dispatch

Solved by ISO in real-time market every 5-15 mins

- Determine injection levels of those units that are online
- Adjustment to dispatch from day-ahead market (unit commitment)

Optimal dispatch

Problem formulation

Model

- Network: graph $G = (\overline{N}, E)$

Optimization vars

- Control:
  - Dispatch: real & reactive power injections $u := (u_j, j \in \overline{N})$
- Network state:
  - Voltages $V := (V_j, j \in \overline{N})$
  - Line flows $S := \left(S_{jk}, S_{kj}, (j,k) \in E\right)$

Optimal dispatch

Problem formulation

Parameters

- Uncontrollable injections $\sigma := \left(\sigma_j, j \in \overline{N}\right)$

Generation cost is quadratic in real power

$$c(u,x) \quad = \sum_{\text{generators } j} \left(a_j \left(\text{Re}(u_j)\right)^2 \; + \; b_j \, \text{Re}(u_j) \right)$$

Optimal dispatch

Constraints

Power flow equations: $S = S(V)$

- Complex form: $S_{jk}(V) \; = \; \left(y_{jk}^s\right)^H \left(|V_j|^2 - V_j V_k^H \right) \; + \; \left(y_{jk}^m\right)^H |V_j|^2$
- Polar form:

$$P_{jk}(V) = \left(g_{jk}^s + g_{jk}^m\right)|V_j|^2 \; - \; |V_j||V_j| \left(g_{jk}^s \cos(\theta_j - \theta_k) \; - \; b_{jk}^s \sin(\theta_j - \theta_k) \right)$$

$$Q_{jk}(V) = \left(b_{jk}^s + b_{jk}^m\right)|V_j|^2 \; - \; |V_j||V_k| \left(b_{jk}^s \cos(\theta_j - \theta_k) \; + \; g_{jk}^s \sin(\theta_j - \theta_k) \right)$$

Power balance: $u_j \; + \; \sigma_j \;\; = \;\; \sum_{k:j\sim k} S_{jk}(V)$

Optimal dispatch

Constraints

Injection limits: $\underline{u}_j \leq u_j \leq \overline{u}_j$

Voltage limits: $\underline{v}_j \leq |V_j|^2 \leq \overline{v}_j$

Line limits: $|S_{jk}(V)| \leq \overline{S}_{jk}, \quad |S_{kj}(V)| \leq \overline{S}_{kj}$

Optimal dispatch

$$\min_{u,x} \quad c(u,x)$$

$$\text{s.t.} \quad u_j + \sigma_j = \sum_{k:j\sim k} S_{jk}(V)$$

$$\underline{u}_j \le u_j \le \overline{u}_j$$

$$\underline{v}_j \le |V_j|^2 \le \overline{v}_j$$

$$|S_{jk}(V)| \le \overline{S}_{jk}, \quad |S_{kj}(V)| \le \overline{S}_{kj}$$

$u^{\text{opt}}(\sigma)$: optimal dispatch driven by σ

Optimal dispatch

Interpretation

- ISO dispatches u_j^{opt} to unit j as generation setpoint (needs incentive compatibility)
- Resulting network state x^{opt} satisfies operational constraints

Economic dispatch in practice

- Real-time market use linear approximation, e.g., DC power flow, instead of AC (nonlinear) power flow equations
- ISO solves linear program for dispatch and wholesale prices
- AC power flow equations are used to verify that operational constraints are satisfied if dispatched
- If not, DC OPF is modified and procedure repeated

Optimal dispatch

Imbalance

In theory, power is balanced at all points of network, since $\left(u^{\text{opt}}, x^{\text{opt}}\right)$ satisfies

$$u_j \ + \ \sigma_j \ \ = \ \sum_{k:j\sim k} S_{jk}(V)$$

Imbalance, however, arises due to

- Random error $\Delta_1(\xi, t)$
- Discretization error $\Delta_2(t)$
- Prediction error $\Delta_3(\xi, t)$

Optimal dispatch

Error model

Uncontrollable injections $\sigma := \left(\sigma(t), t \in \mathbb{R}_+\right)$: continuous-time stochastic process

- Mean process $m(t) := E\sigma(t)$

$u\left(\sigma(\xi, t)\right)$: actual injections that can maintain power balance over network

Imbalance:

$$\Delta u(\xi, t) \;:=\; u\left(\sigma(\xi, t)\right) - u^{\text{opt}}\left(\hat{m}(n)\right), \quad t \in [n\delta, (n+1)\delta),\; n = 0,1,\ldots$$

actual injection at time t

dispatch on nth control interval

- $u\left(\sigma(\xi, t)\right)$: random, continuous
- $u^{\text{opt}}\left(\hat{m}(n)\right)$: fixed for nth interval, based on estimate $\hat{m}(n)$ of σ

Optimal dispatch

Error model

Random error $\Delta_1(\xi, t) \;:=\; u(\sigma(\xi, t)) - u^{\text{opt}}(m(t))$

- Dispatch driven by mean process, at all (continuous) time t

Discretization error $\Delta_2(t) \;:=\; u^{\text{opt}}(m(t)) - u^{\text{opt}}(\bar{m}(n)),\; t \in [n\delta, (n+1)\delta)$

- Dispatch driven by time-average over nth interval $\bar{m}(n) \;:=\; \frac{1}{\delta} \int_{n\delta}^{(n+1)\delta} m(t)\, dt$

Prediction error $\Delta_3(\xi, t) \;:=\; u^{\text{opt}}\left(\bar{m}(n)\right) - u^{\text{opt}}\left(\hat{m}(n)\right),\; t \in [n\delta, (n+1)\delta)$

- Dispatch driven by estimate $\hat{m}(n)$ of $\bar{m}(n)$ before beginning of nth interval
- $\hat{m}(n)$ generally depends on ξ and is random, e.g., avg injection in $n-1$st interval

$$\hat{m}(\xi, n) \;:=\; \frac{1}{\delta} \int_{(n-1)\delta}^{n\delta} \sigma(\xi, t)\, dt$$

Optimal dispatch
Error model

Imbalance:

$$\Delta u(\xi, t) \quad = \quad \Delta_1(\xi, t) \; + \; \Delta_2(t) \; + \; \Delta_3(\xi, t)$$

- Random error $\Delta_1(\xi, t)$: tends to have zero mean
- Discretization error $\Delta_2(t)$: time avg over control interval tends to be small
- Prediction error $\Delta_3(\xi, t)$: tends to be small if $\sigma(t)$ is slow-varying

Outline

1. Overview
2. Unit commitment
3. Optimal dispatch
4. Frequency control
    - Model and assumptions
    - Primary frequency control
    - Secondary frequency control
5. System security

Frequency control

Overview

Power delivered by thermal generator is determined by mechanical output of turbine

- Mechanical output of turbine controlled by opening or closing of valves that regulate steam or water flow
- If load increases, valves will be opened wider to generate more power to balance

Frequency control

Overview

Power delivered by thermal generator is determined by mechanical output of turbine

- Mechanical output of turbine controlled by opening or closing of valves that regulate steam or water flow
- If load increases, valves will be opened wider to generate more power to balance

Power imbalance $\Longrightarrow$ frequency deviates from nominal

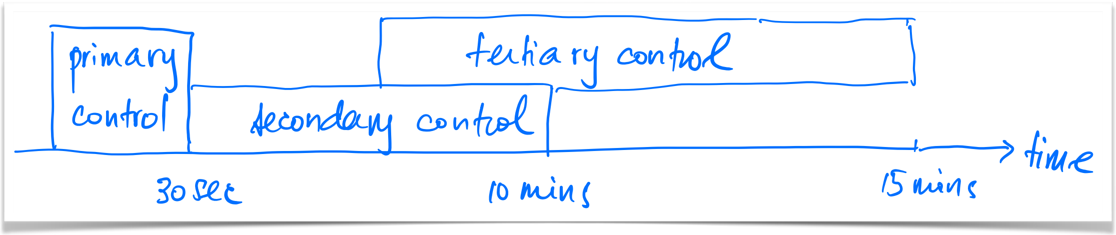
- Excess supply: rotating machines speed up $\Rightarrow$ frequency rises
- Shortage: rotating machines slow down $\Rightarrow$ frequency drops
- If power is not re-imbalanced, frequency excursion will continue and may disconnect generators to protect them from damage
- Can lead to load shedding (blackout) or even system collapse

Frequency control
Overview

Frequency deviation is global control signal for participating generators and loads

Automatic generation control (AGC) : hierarchical control

- **Primary (droop) control**: stabilize frequency in ~30 secs
  - Uses governor to adjust valve position and control mechanical output of turbine
  - Control proportional to local frequency deviation
  - Decentralized



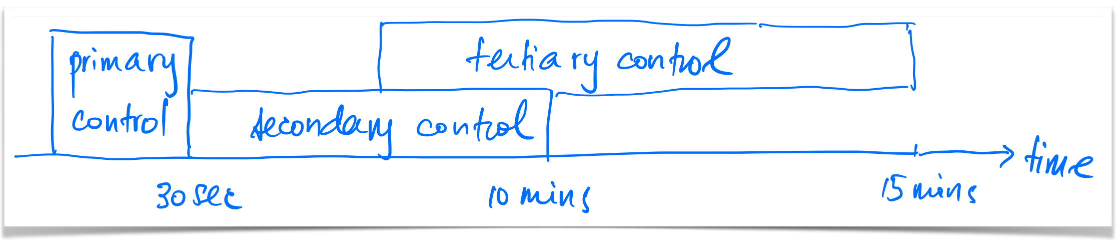
Frequency control

Overview

Frequency deviation is global control signal for participating generators and loads

Automatic generation control (AGC) : hierarchical control

- **Secondary control**: restore nominal frequency within a few mins
  - Adjust generator setpoints around dispatch values
  - Interconnected system: also restore scheduled tie-line flows between areas (need non-local info of tie-line flow deviations)
  - Each area is controlled centrally by an operator



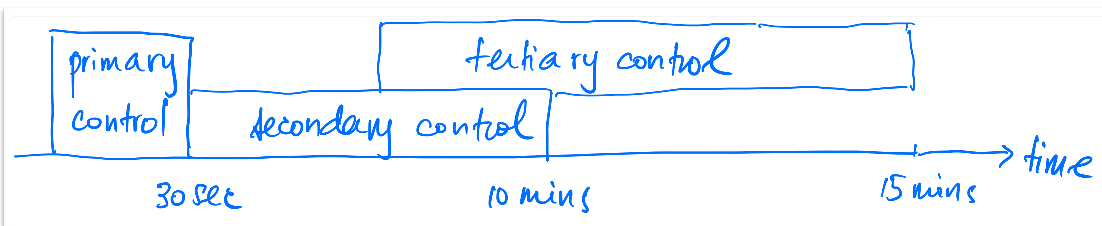
Frequency control

Overview

Frequency deviation is global control signal for participating generators and loads

Automatic generation control (AGC) : hierarchical control

- **Tertiary control**: real-time optimal dispatch every 5-15 mins
  - Determine generator setpoints and schedule inter-area tie-line flows
  - Optimize across areas for economic efficiency
  - Restore reserve capacities of primary & secondary control so that they are available for contingency response



Frequency control
Model

Primary and secondary control model

- Fix control interval n
- Fix random realization ξ of $\sigma(t)$

Assumptions (DC power flow)

- Lossless lines $y_{jk}^s = ib_{jk}$
- Fixed voltage magnitudes (voltage control operates at faster timescale)
- Small angle difference $\sin\left(\theta_{jk}\right) \approx \theta_{jk}$

$\Longrightarrow$ Linearized dynamic model on

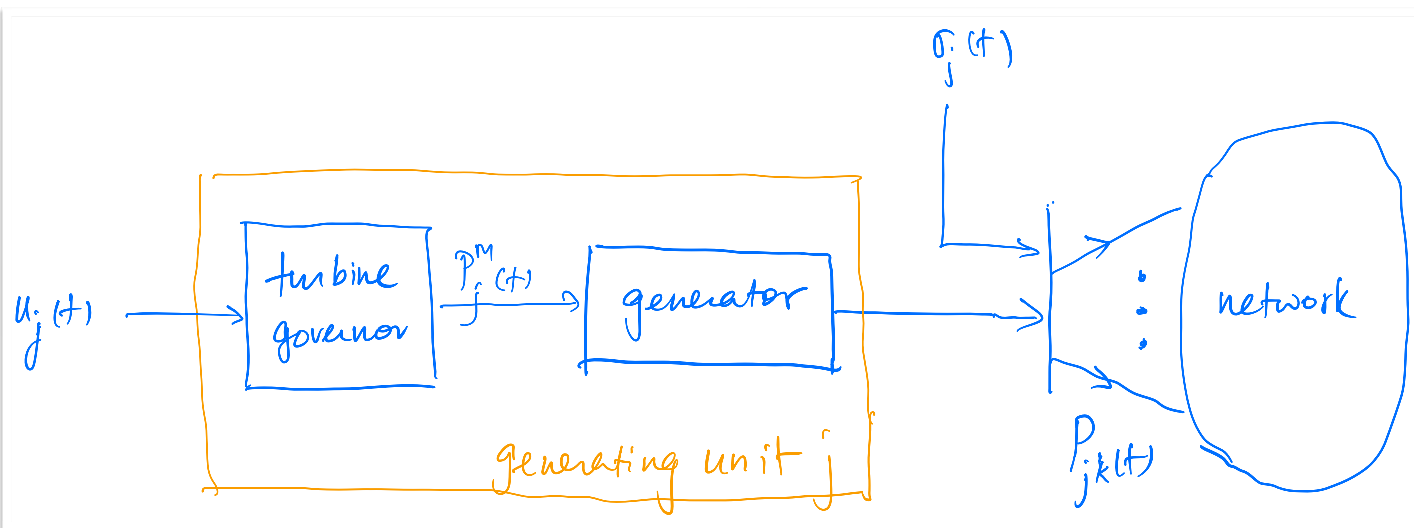
- How real power control voltage angles & local frequencies (derivatives)

Frequency control

Model

Linearized around operating point, defined by

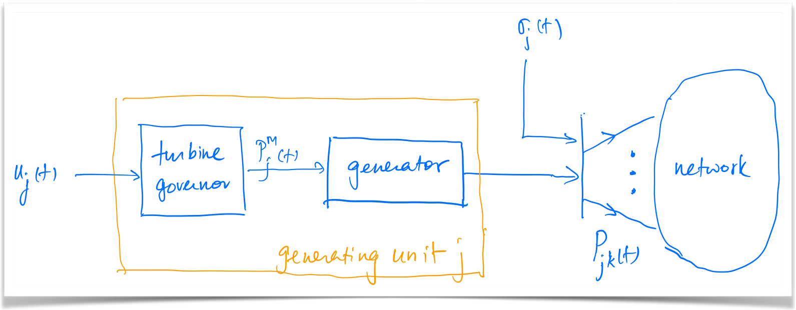
$$u_j^0 \ + \ \sigma_j^0 \ = \ \sum_{k:j\sim k} P_{jk}^0$$



Primary frequency control

Turbine-governor model

2nd order model with droop control



$$T_{gj}\,\dot{a}_j = -a_j(t) + u_j(t) - \frac{\Delta\omega_j(t)}{R_j}$$

$$T_{tj}\,\dot{p}_j^M = -p_j^M(t) + a_j(t)$$

where

- $a_j(t)$: valve position of turbine-governor
- $p_j^M(t)$: mechanical power output of turbine
- $u_j(t)$: generator setpoint (operating point u_j^0 is from tertiary control)
- $\Delta\omega_j(t) = \Delta\dot{\theta}_j(t)$: frequency deviation from operating-point frequency ω^0

Primary frequency control

Turbine-governor model

Linearized around operating point

$$T_{gj}\,\Delta\dot{a}_j = -\,\Delta a_j(t) \;+\; \Delta u_j(t) \;-\; \frac{\Delta\omega_j(t)}{R_j}$$

$$T_{tj}\,\Delta\dot{p}_j^M = -\,\Delta p_j^M(t) \;+\; \Delta a_j(t)$$

incremental vars:

- $\Delta a_j(t) \;:=\; a_j(t) - a_j^0$: deviation of valve position of turbine-governor
- $\Delta p_j^M(t) \;:=\; p_j^M(t) - P_j^{M0}$: deviation of mechanical power output of turbine
- $\Delta u_j(t) \;:=\; u_j(t) - u_j^0$: adjustment to dispatched setpoint

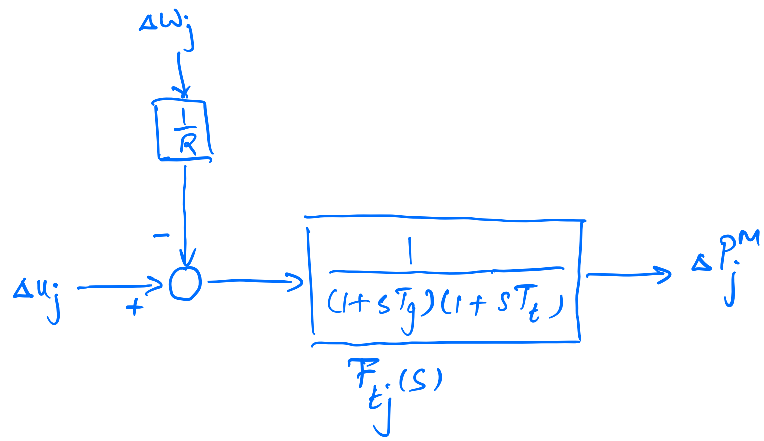
Primary frequency control

Turbine-governor model

Linearized around operating point

$$T_{gj}\, \Delta \dot{a}_j = -\,\Delta a_j(t) \;+\; \Delta u_j(t) \;-\; \frac{\Delta \omega_j(t)}{R_j}$$

$$T_{tj}\, \Delta \dot{p}_j^M = -\,\Delta p_j^M(t) \;+\; \Delta a_j(t)$$



Primary frequency control

Turbine-governor model

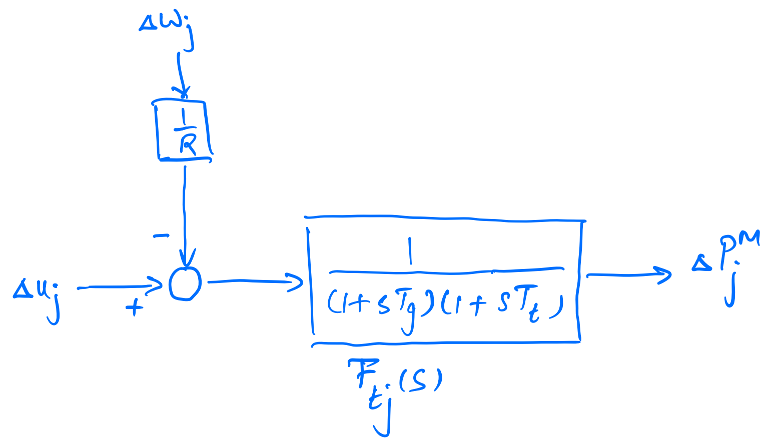
Linearized around operating point

$$T_{gj}\,\Delta\dot{a}_j = -\,\Delta a_j(t) \;+\; \Delta u_j(t) \;-\; \frac{\Delta\omega_j(t)}{R_j}$$

$$T_{tj}\,\Delta\dot{p}_j^M = -\,\Delta p_j^M(t) \;+\; \Delta a_j(t)$$

For primary control, $\Delta u_j(t) = \Delta u_j$ is constant

- $\Delta u_j(t)$ is adjusted by secondary control on a slower timescale



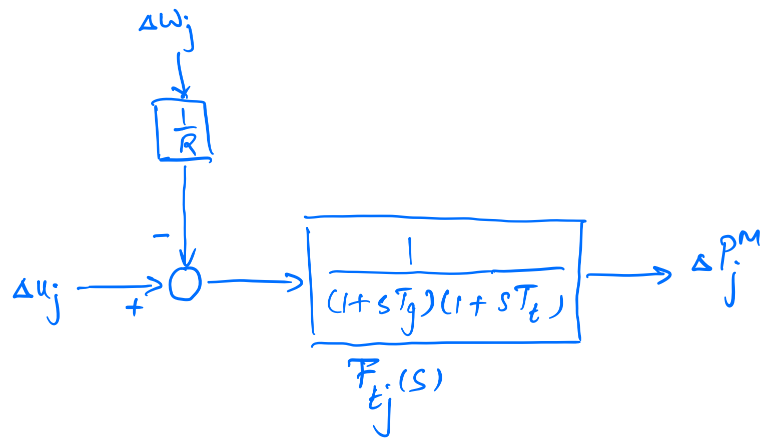
Primary frequency control

Turbine-governor model

Linearized around operating point

$$T_{gj}\,\Delta\dot{a}_j = -\,\Delta a_j(t) \;+\; \Delta u_j(t) \;-\; \frac{\Delta\omega_j(t)}{R_j}$$

$$T_{tj}\,\Delta\dot{p}_j^M = -\,\Delta p_j^M(t) \;+\; \Delta a_j(t)$$



Equilibrium of turbine-governor (primary control):

$$\Delta\dot{a}_j(t) \;=\; \Delta\dot{p}_j^M \;=\; 0$$

Therefore

$$\Delta p_j^{M*} \;=\; \Delta a_j^* \;=\; \Delta u_j \;-\; \frac{1}{R_j}\Delta\omega_j^*,$$

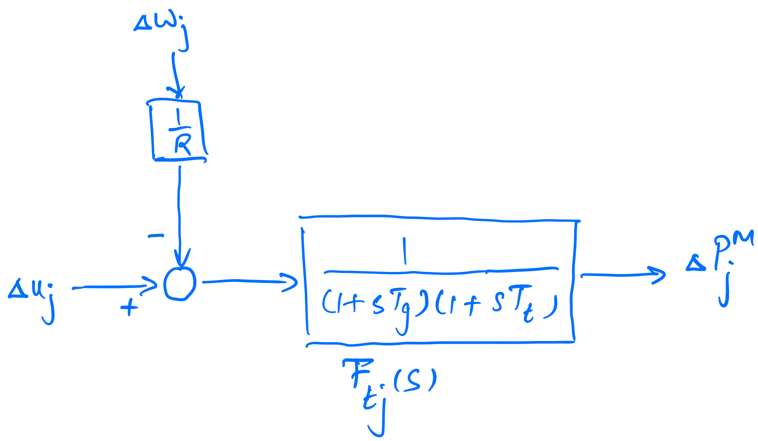
Primary frequency control

Turbine-governor model

Linearized around operating point

$$T_{gj}\, \Delta \dot{a}_j = -\Delta a_j(t) + \Delta u_j(t) - \frac{\Delta \omega_j(t)}{R_j}$$

$$T_{tj}\, \Delta \dot{p}_j^M = -\Delta p_j^M(t) + \Delta a_j(t)$$



Equilibrium of turbine-governor (primary control):

- Frequency deviation $\Delta\omega_j^* \neq 0$
- Incremental mechanical power Δp_j^{M*} depends on $\Delta\omega_j^*$

Primary frequency control

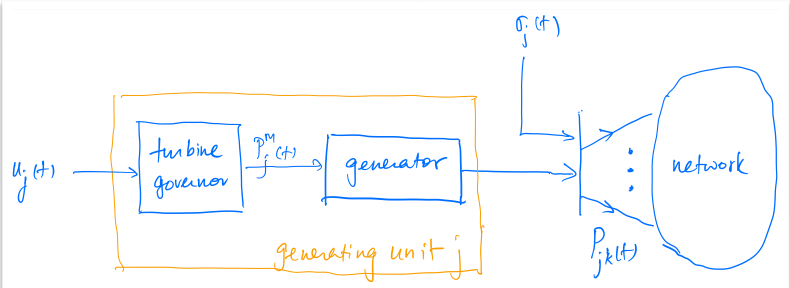
Generator model

$$\Delta\dot{\theta}_j = \Delta\omega_j(t)$$

$$M_j\Delta\dot{\omega}_j + D_j\Delta\omega_j(t) = \Delta p_j^M(t) + \Delta\sigma_j(t) - \sum_{k:j\sim k}\Delta P_{jk}(t)$$

where

- $\Delta\theta_j(t) := \theta_j(t) - \theta_j^0$: incremental angle relative to rotating frame of ω^0
- $\Delta\sigma_j(t)$: deviation of uncontrollable injection from its forecast σ_j^0
- $\Delta P_{jk}(t) := P_{jk}(t) - P_{jk}^0$: line flow deviation



Primary frequency control

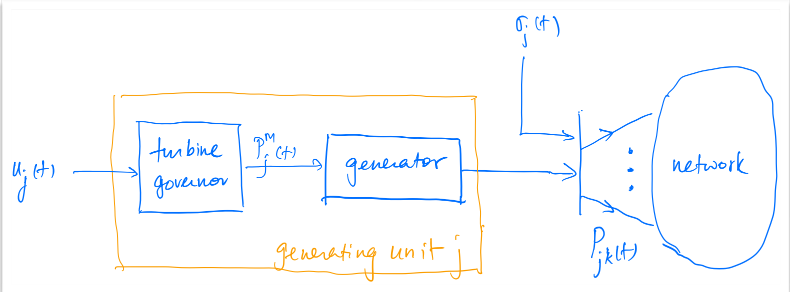
Generator model

$$\Delta\dot{\theta}_j = \Delta\omega_j(t)$$

$$M_j\Delta\dot{\omega}_j + D_j\Delta\omega_j(t) = \Delta p_j^M(t) + \Delta\sigma_j(t) - \sum_{k:j\sim k} \Delta P_{jk}(t)$$

where

- M_j : inertia constant of synchronous machine
- D_j : damping and frequency-sensitive load



Primary frequency control

Generator model

Model for instantaneous line flow

$$P_{jk}(t) \; = \; |V_j|\,|V_k| \left(-b_{jk}\right) \sin\left(\theta_j(t) - \theta_k(t)\right)$$

Primary frequency control

Generator model

Model for instantaneous line flow

$$P_{jk}(t) \;=\; |V_j|\,|V_k| \left(-b_{jk}\right) \sin\left(\theta_j(t) - \theta_k(t)\right)$$

Linear approximation

$$P_{jk}(t) \;=\; \underbrace{|V_j|\,|V_k| \left(-b_{jk}\right) \sin\left(\theta_j^0 - \theta_k^0\right)}_{P_{jk}^0} + \; T_{jk}\left(\Delta\theta_j(t) - \Delta\theta_k(t)\right)$$

Primary frequency control
Generator model

Model for instantaneous line flow

$$P_{jk}(t) \;=\; |V_j||V_k|\left(-b_{jk}\right)\sin\left(\theta_j(t)-\theta_k(t)\right)$$

Linear approximation

$$P_{jk}(t) \;=\; \underbrace{|V_j||V_k|\left(-b_{jk}\right)\sin\left(\theta_j^0-\theta_k^0\right)}_{P_{jk}^0} \;+\; T_{jk}\left(\Delta\theta_j(t)-\Delta\theta_k(t)\right)$$

Linearized model

$$\Delta P_{jk}(t) \;=\; T_{jk}\left(\Delta\theta_j(t)-\Delta\theta_k(t)\right)$$

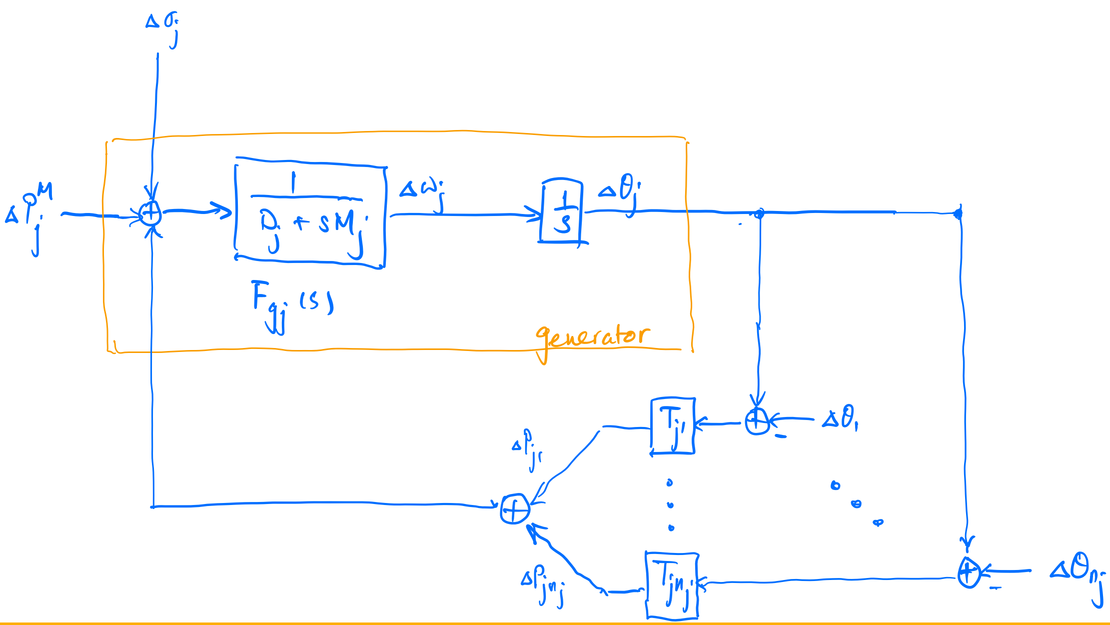
where $T_{jk} := |V_j||V_k|\left(-b_{jk}\right)\cos\left(\theta_j^0-\theta_k^0\right)$

Primary frequency control

Generator model

$$\Delta\dot{\theta}_j = \Delta\omega_j(t)$$

$$M_j\Delta\dot{\omega}_j + D_j\Delta\omega_j(t) = \Delta p_j^M(t) + \Delta\sigma_j(t) - \sum_{k:j\sim k} \Delta P_{jk}(t)$$



Primary frequency control

Turbine-governor-generator model

$$T_{gj}\,\Delta\dot{a}_j = -\,\Delta a_j(t) \;+\; \Delta u_j(t) \;-\; \frac{\Delta\omega_j(t)}{R_j}$$

$$T_{tj}\,\Delta\dot{p}_j^M = -\,\Delta p_j^M(t) \;+\; \Delta a_j(t)$$

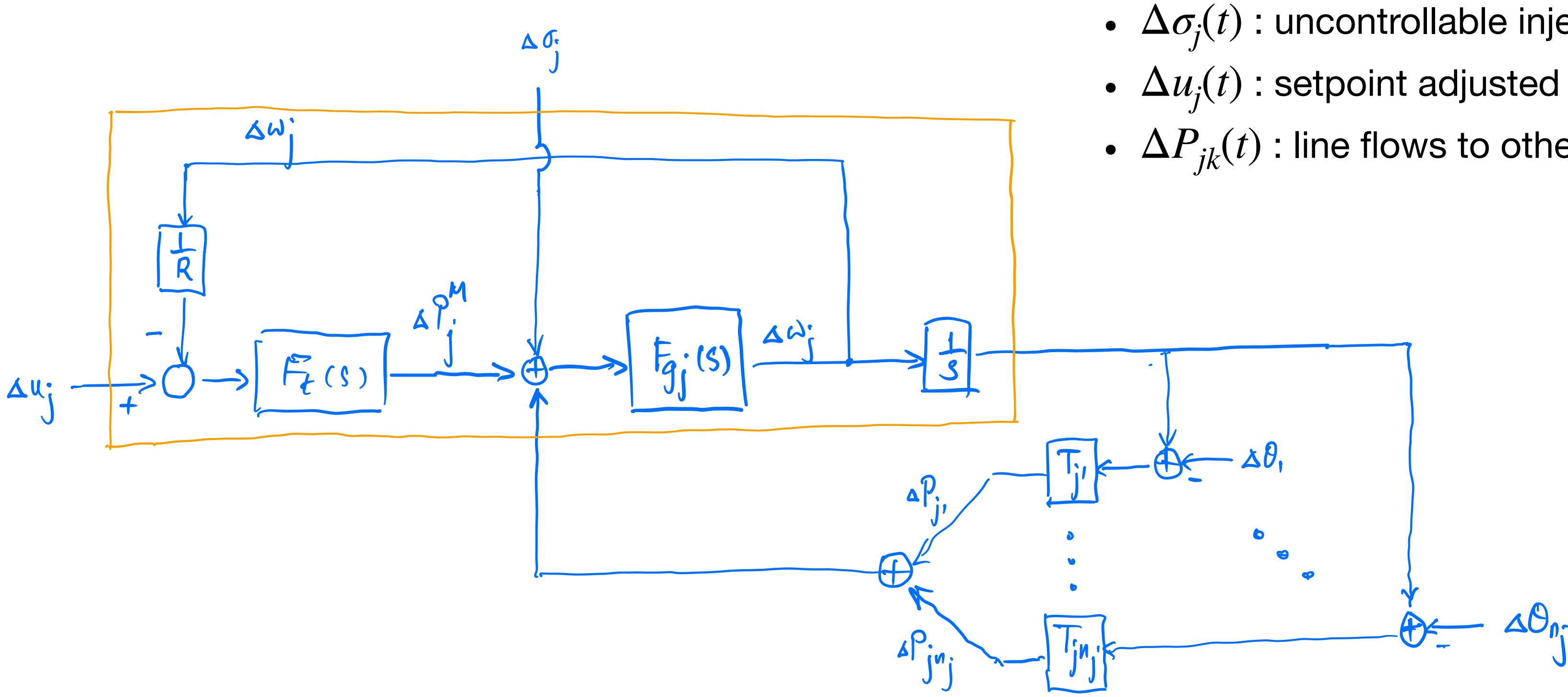
$$M_j\Delta\dot{\omega}_j \;+\; D_j\Delta\omega_j(t) = \;\Delta p_j^M(t) \;+\; \Delta\sigma_j(t) \;-\; \sum_{k:j\sim k}\Delta P_{jk}(t)$$

$$\Delta P_{jk}(t) = \;T_{jk}\left(\Delta\theta_j(t) - \Delta\theta_k(t)\right)$$

$$\Delta\dot{\theta}_j = \;\Delta\omega_j(t)$$

Primary frequency control

Turbine-governor-generator model



Input:

- $\Delta\sigma_j(t)$: uncontrollable injection
- $\Delta u_j(t)$: setpoint adjusted by secondary control
- $\Delta P_{jk}(t)$: line flows to other areas

Primary frequency control

Turbine-governor-generator model

$$T_{gj}\,\Delta\dot{a}_j = -\,\Delta a_j(t) \;+\; \Delta u_j(t) \;-\; \frac{\Delta\omega_j(t)}{R_j}$$

$$T_{tj}\,\Delta\dot{p}_j^M = -\,\Delta p_j^M(t) \;+\; \Delta a_j(t)$$

$$M_j\Delta\dot{\omega}_j \;+\; D_j\Delta\omega_j(t) = \;\Delta p_j^M(t) \;+\; \Delta\sigma_j(t) \;-\; \sum_{k:j\sim k}\Delta P_{jk}(t)$$

$$\Delta P_{jk}(t) = \;T_{jk}\left(\Delta\theta_j(t) - \Delta\theta_k(t)\right)$$

$$\Delta\dot{\theta}_j = \;\Delta\omega_j(t)$$

Equilibrium of primary control: $\Delta\dot{\omega}_j \;=\; \Delta\dot{a}_j \;=\; \Delta\dot{p}_j^M \;=\; 0$ (does not require $\Delta\dot{\theta} = 0$)

Primary frequency control

Equilibrium

Bus-by-line incidence matrix C :

$$C_{jl} \;:=\; \begin{cases} 1 & \text{if } \; l = j \rightarrow k \text{ for some bus } k \\ -1 & \text{if } \; l = i \rightarrow j \text{ for some bus } i \\ 0 & \text{otherwise} \end{cases}$$

Stiffness matrix: $T \;:=\; \text{diag}(T_{jk}, (j,k) \in E)$

Laplacian matrix: $L \;:=\; CTC^T$ and its pseudo-inverse $L^\dagger$

Primary frequency control

Equilibrium

Theorem

Let $x^* \;:=\; \left(\Delta\omega^*, \Delta P^*, \Delta\theta^*, \Delta a^*, \Delta p^{M*}\right)$ be an equilibrium driven by step change $\Delta\sigma$ and constant setpoint Δu

Primary frequency control

Equilibrium

Theorem

Let $x^* \ := \ \left(\Delta\omega^*, \Delta P^*, \Delta\theta^*, \Delta a^*, \Delta p^{M*}\right)$ be an equilibrium driven by step change $\Delta\sigma$ and constant setpoint Δu

1. Local frequency deviations converge to

$$\Delta\omega_j^* \quad = \quad \Delta\omega^* \quad := \quad \frac{\sum_k \left(\Delta u_k + \Delta\sigma_k\right)}{\sum_k \left(D_k + 1/R_k\right)}$$

Primary frequency control

Equilibrium

Theorem

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2. Line flow deviations converge to

$$\Delta P^* \;=\; TC^TL^\dagger \left(\Delta u \;+\; \Delta\sigma \;-\; \Delta\omega^* d\right)$$

where $d \;:=\; (D_j + 1/R_j, j \in \overline{N})$

Primary frequency control

Equilibrium

Theorem

Let $x^* := \left(\Delta\omega^*, \Delta P^*, \Delta\theta^*, \Delta a^*, \Delta p^{M*}\right)$ be an equilibrium driven by step change $\Delta\sigma$ and constant setpoint Δu

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where $d := (D_j + 1/R_j, j \in \overline{N})$

Secondary control:

- Adjusting Δu to drive $\Delta\omega_j^*$ and ΔP_{jk}^* to 0

Primary frequency control

Example: interconnected system

Model

- $N + 1$ areas each modeled as a bus
- $\Delta u_j = 0$ for all j
- Step change: at time 0, $\sigma_j(t)$ changes from 0 to a constant value $\Delta\sigma_j$
- Suppose $\Delta\sigma_j$ are iid random variables with mean $\Delta\bar{\sigma}_j$ and variance v_j^2

Compare the mean & variance of equilibrium frequency deviation $\Delta\omega_j^*$:

- Case 1: the areas (buses) are not connected and operate independently.
- Case 2: the areas (buses) are connected into a network

Primary frequency control

Example: interconnected system

Case 1: independent operation

$$\Delta\omega_j^* \;=\; \frac{\Delta\sigma_j}{d_j} \qquad \text{where } d_j := D_j + 1/R_j$$

with $\quad E\Delta\omega_j^* \;=\; \dfrac{\Delta\bar{\sigma}_j}{d_j}, \quad \text{var}\left(\Delta\omega_j^*\right) \;=\; \dfrac{v_j^2}{d_j^2}$

Case 2: interconnected system

$$\Delta\omega^* \;=\; \frac{\sum_j \Delta\sigma_j}{\sum_j d_j} \;=\; \frac{1}{N+1}\sum_j \frac{\Delta\sigma_j}{\hat{d}} \qquad \text{where } \hat{d}_j := \frac{1}{N+1}\sum_j d_j$$

with $\quad E\Delta\omega^* \;=\; \dfrac{\Delta\hat{\sigma}}{\hat{d}}, \quad \text{var}(\Delta\omega^*) \;=\; \dfrac{1}{N+1}\dfrac{\hat{v}^2}{\hat{d}^2} \quad$ where $\Delta\hat{\sigma},\ \hat{v}^2$ are avgerages

Frequency control
Model

Linearized around operating point, defined by

$$u_j^0 \ + \ \sigma_j^0 \ \ = \ \sum_{k:j\sim k} P_{jk}^0$$

Incremental variables (full list)

- $\Delta u_j(t) \ := \ u_j(t) - u_j^0$: adjustment to dispatched setpoint
- $\Delta \theta_j(t) \ := \ \theta_j(t) - \theta_j^0$: incremental angle relative to rotating frame of ω^0
- $\Delta \omega_j(t) = \Delta \dot{\theta}_j(t)$: frequency deviation from operating-point frequency ω^0
- $\Delta P_{jk}(t) \ := \ P_{jk}(t) - P_{jk}^0$: line flow deviation
- $\Delta p_j^M(t) \ := \ p_j^M(t) - P_j^{M0}$: deviation of mechanical power output of turbine
- $\Delta a_j(t) \ := \ a_j(t) - a_j^0$: deviation of valve position of turbine-governor

Outline

1. Overview
2. Unit commitment
3. Optimal dispatch
4. Frequency control
    - Model and assumptions
    - Primary frequency control
    - Secondary frequency control
5. System security

Secondary frequency control

Objectives

1. Restore frequency to nominal value
   - Drive $\Delta\omega^* = 0$
2. Restore tie-line flows to scheduled values (scheduled by tertiary control)
   - Drive $\Delta P^* = 0$ (each bus represents a control area)

Secondary frequency control

Objectives

At equilibrium of primary control :

$$\Delta\omega_j^* \quad = \quad \Delta\omega^* \quad := \quad \frac{\sum_k \left(\Delta u_k + \Delta\sigma_k\right)}{\sum_k \left(D_k + 1/R_k\right)}$$

$$\Delta P^* \quad = \quad TC^T L^\dagger \left(\Delta u \ + \ \Delta\sigma \ - \ \Delta\omega^* d\right)$$

Therefore, need to adjust setpoints $\Delta u(t)$

- $\Delta\omega_j^* = 0$ if $\sum_k \left(\Delta u_k + \Delta\sigma_k\right) = 0$
- $\Delta P_{jk}^* = 0$ if $\Delta u_j + \Delta\sigma_j = 0$

Secondary frequency control

Area control error (ACE)

$$\text{ACE}_j(t) \quad := \quad \sum_{k:j\sim k} \Delta P_{jk}(t) \quad + \quad \beta_j \Delta\omega_j(t)$$

Setpoint adjustment

$$\Delta \dot{u}_j \;= -\gamma_j \left(\sum_{k:j\sim k} \Delta P_{jk}(t) \quad + \quad \beta_j \Delta\omega_j(t) \right)$$

Implementation

- Real-time measurements of $P_{jk}(t)$ with neighboring areas k are sent to system operator
- System operator centrally computes $\Delta \dot{u}_j$ and dispatch setpoint adjustments $\alpha_{ji}\Delta u_j(t)$ to participating generators i in areal j ($\alpha_{ji} \geq 0$ with $\sum_i \alpha_{ji} = 1$ are called participation factors)

Secondary frequency control

Overall (primary & secondary) model

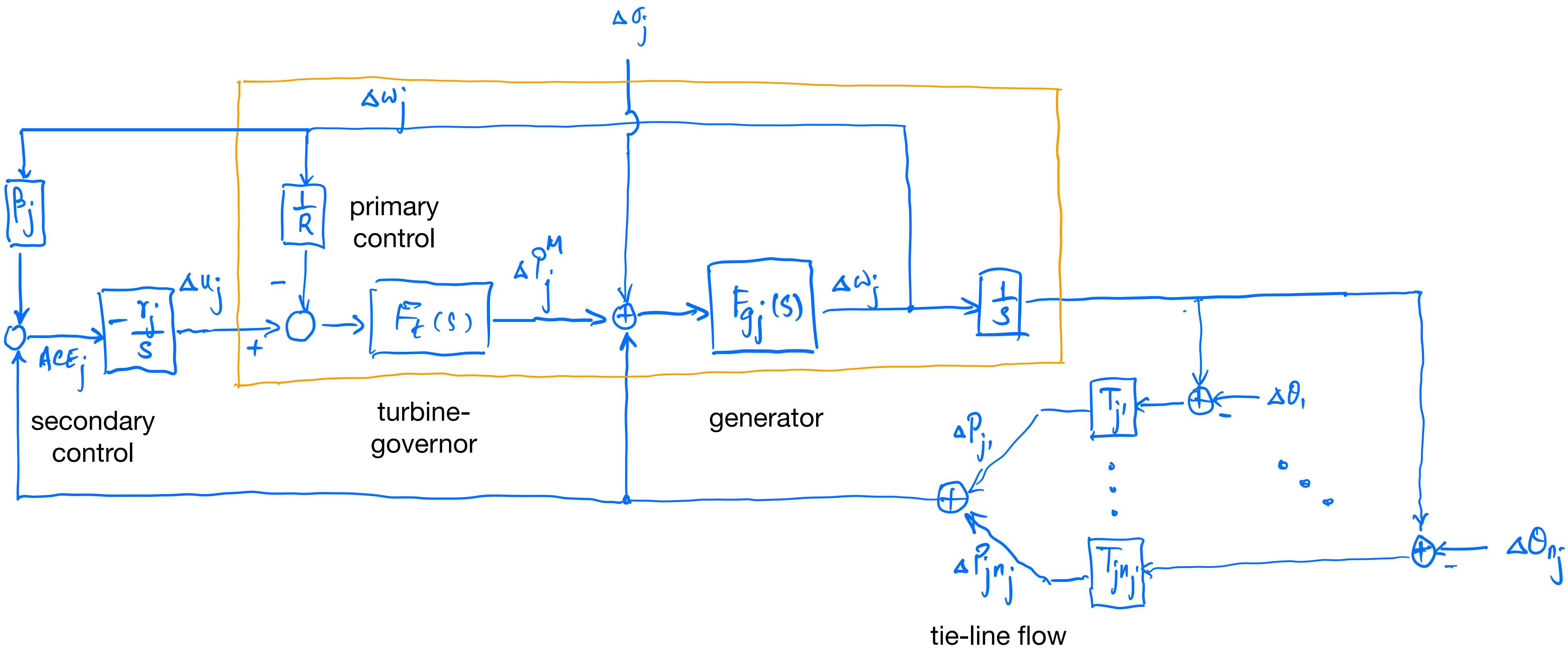
$$
\left.\begin{aligned}
T_g \, \Delta \dot{a} &= -\Delta a(t) + \Delta u(t) - R^{-1} \Delta\omega(t) \\
T_t \, \Delta \dot{p}^M &= -\Delta p^M(t) + \Delta a(t)
\end{aligned}\right\} \text{turbine-governor}
$$

$$
\left.\begin{aligned}
M \Delta\dot{\omega} + D \Delta\omega(t) &= \Delta p^M(t) + \Delta\sigma(t) - C \Delta P(t) \\
\Delta P(t) &= T C^T \Delta\theta(t) \\
\Delta\dot{\theta} &= \Delta\omega(t) \\
\Delta\dot{u} &= -\Gamma \left(C \Delta P(t) + B \Delta\omega(t) \right)
\end{aligned}\right\} \text{generator}
$$

Equilibrium of secondary control: $\Delta\dot{u} = \Delta\dot{\omega} = \Delta\dot{a} = \Delta\dot{p}^M = 0$
(does not req $\Delta\dot{\theta} = 0$)

Secondary frequency control

Overall (primary & secondary) model



Secondary frequency control

Equilibrium

Theorem

Let $x^* \;:=\; \left(\Delta u^*, \Delta\omega^*, \Delta P^*, \Delta\theta^*, \Delta a^*, \Delta p^{M*}\right)$ be an equilibrium driven by step change $\Delta\sigma$

1. Frequencies are restored to ω^0 : $\Delta\omega^* = 0$

Secondary frequency control

Equilibrium

Theorem

Let $x^* \ := \ \left(\Delta u^*, \Delta\omega^*, \Delta P^*, \Delta\theta^*, \Delta a^*, \Delta p^{M*}\right)$ be an equilibrium driven by step change $\Delta\sigma$

1. Frequencies are restored to ω^0 : $\Delta\omega^* = 0$
2. Line flow are restored to P^0 : $\Delta P^* = 0$

Secondary frequency control

Equilibrium

Theorem

Let $x^* \;:=\; \left(\Delta u^*, \Delta\omega^*, \Delta P^*, \Delta\theta^*, \Delta a^*, \Delta p^{M*}\right)$ be an equilibrium driven by step change $\Delta\sigma$

1. Frequencies are restored to ω^0 : $\Delta\omega^* = 0$
2. Line flow are restored to P^0 : $\Delta P^* = 0$
3. Disturbances are compensated for locally at each bus (i.e., in each area) : $\Delta u_j^* + \Delta\sigma_j = 0$

Outline

1. Overview
2. Unit commitment
3. Optimal dispatch
4. Frequency control
5. System security

System security

- System security refers to ability to withstand contingency events
- A contingency event is an outage of a generator, transmission line, or transformer
- Contingency events are rare, but can be catastrophic
- NERC's (North America Electricity Reliability Council) $N-1$ rule the outage of a single piece of equipment should not result in violation of voltage or line limits

System security

Secure operation

- Analyze credible contingencies that may lead to voltage or line limit violations
- Account for these contingencies in optimal commitment and dispatch schedules (security constrained UC/ED)
- Monitor system state in real time and take corrective actions when contingency arises

Optimal dispatch

Recall: OPF without security constraints (base case):

$$\min_{(u_0, x_0)} \quad c_0\left(u_0, x_0\right)$$

$$\text{s.t.} \quad f_0\left(u_0, x_0\right) \ = \ 0, \ \ g_0\left(u_0, x_0\right) \ \leq \ 0$$

where

- u_0 : dispatch in base case
- x_0 : network state in base case
- $f_0\left(u_0, x_0\right)$: power flow equations, etc.
- $g_0\left(u_0, x_0\right)$: operational constraints

Security constrained OPF

Preventive approach

Basic idea

- Augment optimal dispatch (OPF) with additional constraints …
- … so that the (new) network state under optimal dispatch u^{opt} will satisfy operational constraints after contingency events
- Dispatch remains unchanged until next update period, even if a contingency occurs in the middle of control interval

Security constrained OPF

Preventive approach

Security constrained OPF (SCOPF)

$$\min_{(u_0, x_0,\ \tilde{x}_k,\ k \geq 1)} \quad c_0\left(u_0, x_0\right)$$

$$\text{s.t.} \quad f_0\left(u_0, x_0\right) = 0, \quad g_0\left(u_0, x_0\right) \leq 0 \quad \text{base case constraints}$$

$$\tilde{f}_k\left(u_0, \tilde{x}_k\right) = 0, \quad \tilde{g}_k\left(u_0, \tilde{x}_k\right) \leq 0 \quad \text{constraints after cont. } k$$

where

- $\tilde{x}_k$: new state under same dispatch u_0 after contingency k
- $\tilde{f}_0\left(u_0, \tilde{x}_0\right)$: power flow equations for post-contingency network
- $\tilde{g}_0\left(u_0, \tilde{x}_0\right)$: (more relaxed) emergency operational constraints after contingency k

Security constrained OPF

Corrective approach

Basic idea

- Compute optimal dispatch not only for base case, but also for each contingency k
- System operator can dispatch a response immediately after contingency without waiting till next dispatch period

Security constrained OPF

Corrective approach

Security constrained OPF (SCOPF)

$$\min_{(u_k, x_k,\ k\geq 0)} \quad \sum_{k\geq 0} w_k\, c_k\left(u_k, x_k\right)$$

$$\text{s.t.} \quad f_k\left(u_k, x_k\right) \ = \ 0, \quad g_k\left(u_k, x_k\right) \ \leq \ 0, \quad k \geq 0$$

$$\|u_k - u_0\| \ \leq \ \rho_k, \quad k \geq 1 \qquad \text{ramp rate limits}$$

where

- $\left(u_k, x_k\right)$: dispatch & state in base case $k = 0$ and after contingency $k \geq 1$
- $\left(f_k, g_k\right)$: power flow equations & operational constraints for $k \geq 0$
- $\|u_k - u_0\|$: ramp rate limits

Conclusion

Central challenge: balance supply & demand second-by-second

- While satisfying operational constraints, e.g. injection/voltage/line limits
- Unlike usual commodities, electricity cannot (yet) be stored in large quantity

This is achieved through a complex set of mechanisms that operate in concert across multiple timescales

- Slow timescale mechanisms (minutes and up) can be formulated as OPF problems
- Fast timescales (seconds to minutes) can be formulated as feedback control problems

Part III of text: OPF

- Mathematical formulations, computational properties, convex relaxations, stochastic optimization